University of Brasília
Mechanical Science Graduate Program

Unit information

Program	Mechanical Science (53001010053P0)
Course unit	Continuum Mechanics
Unit code	PCMEC
Unit number	0160
Credit points	4
Period	01/01/2004 - Current
Professor	Francisco Ricardo da Cunha
Prerequisites	No. Preliminary knowledge on linear algebra, solid mechanics and fluid mechanics is recomendable, but it is not a requirement.

Unit outline

Objective:

This theoretical course aims to introduce in a unified way the fundamentals and basic principles of Continuum Mechanics (CM) applied to the modeling of deformable solids and fluids in motion. The central idea in CM is to define local average quantities such as density, displacement, velocity and energy as continuous functions in space and time, assuming matter as a continuous medium. For this end, the following fundamental topics will be addressed throughout the course: (i) elements of tensor algebra; (ii) tensor calculus elements; (iii) deformation and kinematics of a continuous medium; (iv) stress; (v) basic laws of the CM: conservation of mass, balance of force, balance of torque torque, balance of energy and principle of entropy; (vi) constitutive formalism and constitutive equations for continuous media with linear and non-linear behavior;
(vii) constitutive equations for the description of membranes and interfaces between continuous media.

Purpose:

Continuum Mechanics (CM) involves the most relevant set of partial differential equations of balance from mechanical sciences and engineering (i.e. mass balance, momentum, angular momentum and energy). These equations describe the motion of solids and fluids when suplemented by the closure material or constitutive equations, involving stress-strain relationship (for solids) or stress-strain rate relationship (for fluids). Solid materials are described in terms of displacement and rotation, while fluids (liquids and gases) in terms of velocity and rate of rotation (i.e. angular velocity or vorticity). Continuum Mechanics connects the properties of continuous materials, whether solid or fluid, to constitutive models in elasticity, viscoelasticity, memory materials, plasticity, complex fluids, electrodynamics and ferrohydrodynamics. There are several applications of Continuum Mechanics, for instance to metals and incompressible viscoelastic materials (rubber). These topics are particularly relevant for students developing research in Mechanical Sciences. Once familiarized with the basic concepts and general principles of the CM course, it is expected that students will not encounter major difficulties in specializing in the aforementioned subareas of solid mechanics and fluid mechanics, in the subsequent stages of their academic training. The course can serve as lectures notes for a graduate level course in continuum mechanics for mechanical science and engineering students.

Contents:

	Module 1 – Tensor algebra. Einstein-Jeffrey index notation, cartesian tensors, operations with tensors, higher order tensors, isotropic tensors, tensors as a linear operator, transpose tensor, symmetric and antisymmetric tensors, dual vector of an antisymmetric tensor, eigenvalues and eigenvectors of a tensor, invariants of a tensor and deviatoric tensors, Cayley-Hamilton theorem, definite positive tensor and theorem of the polar decomposition; **Module 2 – Calculus of tensors**. scalar, vector and tensor functions, differentiation, differential operators, Integral theorems and variations, Helmholtz integral representation; **Module 3 – The continuum hypothesis**. Definition of a continuum media, molecular versus continuum description, configuration of a continuum, motion description in material and spatial coordinates; **Module 4 – Kinematics of a continuum: motion and deformation.** Deformation gradient tensor, Arcs, surfaces and volumes transformations, elongation and rotation, Green, Almansi and Euler deformation tensors, material and spatial derivatives, time derivatives with rotation and deformation: Jaumann and Oldroyd, strain rate and rate rotation (vorticity), Jacobian, divergent and dilation, Transport theorem: integral formulation, mass balance (spatial and material description). **Module 5 – Stress**. Body forces and surface forces, normal stresses, shear stress and principal stresses, principal axes of stress and isotropy, free surfaces: boundary conditions, Piola-Kirchhoff (P-K) tensors, membrane dynamics: tensor formulation, curvature and bending moment; **Module 6 – Fundamental laws of Continuum Mechanics.** Cauchy principle, force balance and Cauchy stress tensor, torque balance and condition of stress tensor symmetry, general equation of non-inertial motion, energy balance, entropy principle (Classius's inequality statement); **Module 7 – Constitutive Formalism and Applications.** Principle of material invariance – (MFI), homogeneity and isotropy, simple Materials: local action and memory, generalized Hooke's law (elastic solid), governing equations of linear elasticity (continuity equation, Navier and energy equations), generalized Newton's law of viscosity, non-Stokesian newtonian fluids, Navier-Stokes equations, models for shear rate dependent viscosity and viscoelastic materials.
Assessment	Two exams (50% of the final grade), self-student exercices (30% of the final grade), Exercise lists (20% of the final grade). Exercise lists will be delivered during the course in order to prepare students for the assessment test.
Obs:	There exist no books covering the whole content of this couse. However the following books below can be recommended.
Reference:	**1)** Aris, R., Vectors, Tensors and the Basic Equations of Fluid Mechanics, Dover, 1962; **2**) Chorlton, F., Ellis Horwood LTD. Vector & Tensor Methods, 1976; **3**) Flugge, W., Tensor Analysis and Continuum Mechanics, Springer Verlag, 1972**.; 4)** Gurtin, M.E., Introduction to Continuous Mechanics, Gurtin, M.E., Academic Press, 1980; **5)** Continuum Mechanics, Chandrasekharaiah, D.S. & Debnath Lokenath, Academic Press, 1994. **6)** Lai, W.M. et al. Introduction to Continuum Mechanics, Elsevier, 2010. **7)** Flugge, S., Truesdell, C. e Noll, The Classical Field Theories, Encyclopedia of Physics, vol. 3-1, Springer 1960.